

Twineham CE Primary School

How we teach calculations:

Calculation Policy for Mathematics

September 2015

Glossary

Composite numbers: Numbers which can be divided by number other than 1 or itself.

Factors: Numbers you can multiply together to get another number.

Four operations: Addition, subtraction, multiplication, division.

Integers: Whole numbers, including negative numbers.

Inverse: Reversing or undoing a calculation e.g. to undo adding you need to subtract.

Non-integer numbers: Numbers which include fractions and decimals.

Prime Numbers: A number which can only be divided by 1 and itself.

Prime Factors: Where a number can be made by only multiplying prime numbers e.g. $12 = 2 \times 2 \times 3$ (2 and 3 are both prime numbers).

The overall aim is that when children leave Twineham in Year 6 they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to carry out calculations mentally apply general strategies and particular strategies (doubling, adding 10 and subtracting 1 to add 9);
- make use of diagrams and jottings to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient and reliable written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;

About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

Age stage expectations

The calculation policy is organised according to year group expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the level that they are currently working at**, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on. Therefore pupils maybe consolidating objectives and strategies from the previous year.

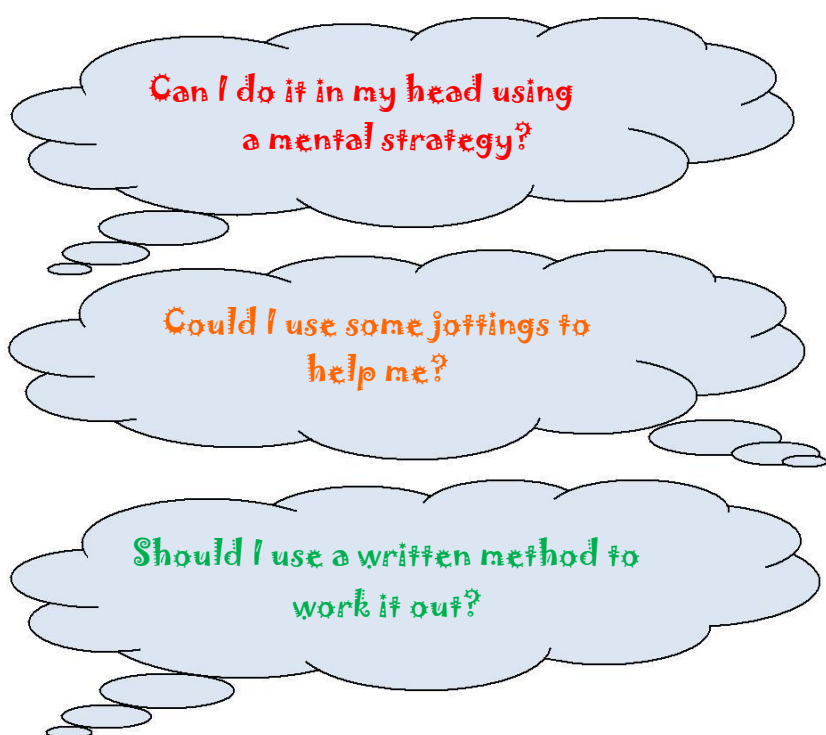
Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems.

Choosing a calculation method:

The methods shown are only written ways of calculating. Children will also be calculating using mental strategies too.

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



To work out a tricky calculation:

Approximate,

Calculate,

Check it!

Addition

EYFS Find one more than a number from 1 to 20

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

Numicon is introduced to support understanding of number.

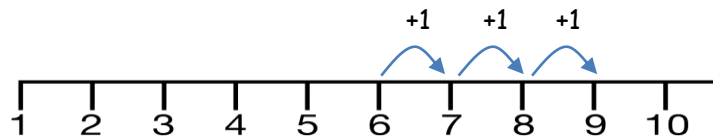
There are three people on the bus. One more gets on. How many are there now?



$$5 + 1 = 6$$

Year 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on. Children also use Numicon to complete calculations and look for addition patterns.

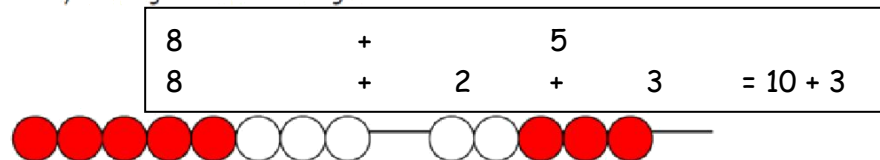


Children should:

- Have access to a wide range of counting equipment, everyday objects and number lines, and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

$$8 + 3 = \square \quad 15 + 4 = \square \quad 5 + 3 + 1 = \square \quad \square + \square = 6 \quad 12 + \square = 16 \quad \square + 4 = 12$$

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line*

Key skills for addition at Y1:

- Read and write numbers to 100 in numerals, incl. 1–20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and beyond 100
- Count in multiples of 1 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations. E.g. Sam has 5 toy cars and is given 6 more. How many does he have?

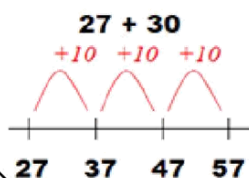
Addition



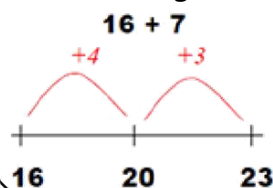
Year 2 Add with 2-digit numbers Developing mental fluency with

addition and place value involving 2-digit numbers, then establish more formal methods.

Add 2-digit numbers and tens:

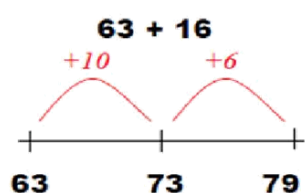


Add 2-digit numbers and units:



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:



	2	0	+	3	
+	3	0	+	4	
<hr/>					
	5	0	+	7	
					= <u>57</u>

STEP 1: Provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

58 + 43:

5	0	+	8
4	0	+	3
<hr/>			
9	0	+	11
<hr/>			
		=	101

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).

Say 50+40 and 8+3

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or Numicon, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary*

Key skills for addition at Y2:

- Add a 2-digit number and a single digit (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Addition

Year 3 Add numbers with up to 3-digits

Focus on being secure with 2 digit numbers before moving to 3 digit numbers

Revise expanded method from Y2 and introduce the **expanded column addition** method:

	2	3	6
+		7	3
<hr/>			
			9
	1	0	0
	2	0	0
<hr/>			
	3	0	9

Add the **units** first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

It is really important that children line up the values carefully.

Move to the compact **column addition** method, with "carrying".

2 3 6

Add **units** first.

+ 7 3

3 0 9

1

"Regroup" numbers underneath the bottom line.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to "regrouping" for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils the actual value is "**three tens** add **seven tens**", not "three add seven", which equals **ten** tens.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary**, increase, vertical, 'regroup', expanded, compact*

Key skills for addition at **Y3**:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- Add a three-digit number and a single digit mentally ($175 + 8$)
- Add a three-digit number and tens mentally ($249 + 50$)
- Add a three-digit number and hundreds mentally ($381 + 400$)
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, and ones.)
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Addition



Year 4 Add numbers with up to 4 digits

Continue to revise expanded methods before moving to the compact column method, **adding units first**, and “regrouping” numbers **underneath** the calculation. Start with 2 digit numbers and gradually increase the size of numbers as children become confident. Also include money and measures contexts.

E.g. $3517 + 396 = 3913$

	3	5	1	7
+		3	9	6
	3	9	1	3

Introduce the **compact column addition method** by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with regrouping, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

“regroup” numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds **not** 5 add 3, for example.

Use and apply this method to money and measurement values.

If using a decimal number make sure the decimal point is lined up in the same column -

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, “regroup”, expanded, compact, **thousands, hundreds, digits, inverse***

Key skills for addition at **Y4**:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context (Sam buys a chocolate bar for 52p and a packet of crisp for 78p. How much change does he get from £2?), deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles (34+35), partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

Addition



Year 5 Add numbers with more than 4 digits

Including money, measures and decimals with different numbers of decimal places.

$$\begin{array}{r} \pounds 42.59 \\ + \pounds 7.67 \\ \hline \pounds 50.26 \end{array}$$

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

The regrouping digit should be smaller than the main

$$\begin{array}{r} 39502 \\ + 8794 \\ \hline 48296 \end{array}$$

Numbers should exceed 4 digits.

$$\begin{array}{r} 19.01 \\ + 3.65 \\ \hline 23.36 \end{array}$$

Pupils should be able to add more than two values, carefully aligning place value columns.

Make sure the decimal point is clear

Say "6 tenths add 7 tenths2 to reinforce place value.

Empty decimal places can be filled with zero to show the place value in each column.

Children should:

- Understand the place value of **tenths** and **hundredths** and use this to align numbers with different numbers of decimal places.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, "regroup", expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths*

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. add the nearest multiple of 10, 100, 1000 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

Addition



Year 6 Add several numbers of increasing complexity

	2	3	.	3	6	1
		9	.	0	8	0
	5	9	.	7	7	0
+		1	.	3	0	0
	9	3	.	5	1	1
	2	1		2		

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places can be filled with zero to show the place value in each column.

	8	1	,	0	5	9
		3	,	6	6	8
		1	5	,	3	0
+		2	0	,	5	5
	1	2	0	,	5	7
		1			1	

Adding several numbers with more than 4 digits.

The decimal point and any commas need to be clearly distinguished

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, "regroup", expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths*

Key skills for addition at Y6:

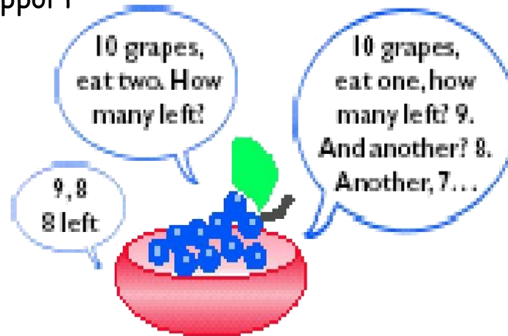
- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Subtraction

EYFS Find one less than a number from 1 to 20

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction. Numicon is also used to support understanding.



They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

$$6 - 2 = 4$$



'Take two apples away. How many are left?'

Subtraction

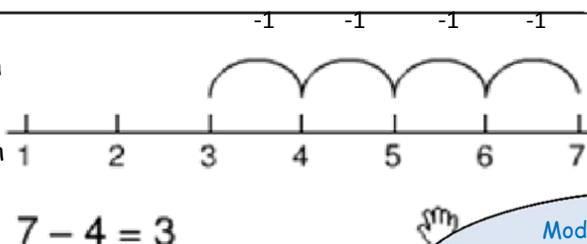
Year 1 Subtract from numbers up to 20

The link between addition and subtractions is established. Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with - and = signs.

Subtract by taking away

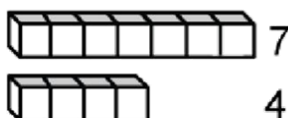
Count back in ones on a numbered number line to take away, with numbers up to 20:



Model subtraction using hundred squares and numbered number lines/tracks practically and with Numicon.

Find the "difference between"

This will be introduced practically with the language "find the difference between" and "how many more?" in a range of familiar contexts.



"Seven is 3 more than four"

"I am 2 years older than my sister"

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero. Children should also start to use the inverse of addition facts.

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?*

Key skills for subtraction at **Y1**:

- Given a number, say **one more** or **one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

Subtraction

Year 2 Subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

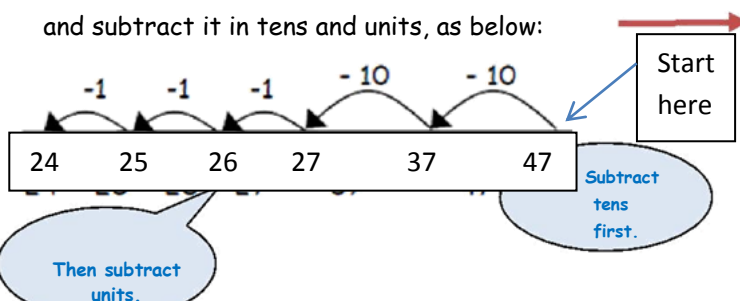
Use Dienes blocks and Numicon for subtraction calculations too

This strategy will be used for:

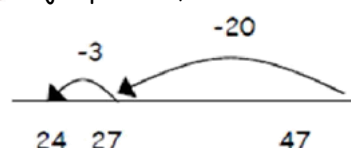
- 2-digit numbers subtract units (by taking away / counting back) e.g. $36-7$
- 2-digit numbers subtract tens (by taking away / counting back) e.g. $48-30$
- Subtracting pairs of 2-digit numbers (see below)

Subtracting pairs of 2-digit numbers on a number line:

$47 - 23 = 24$ Partition the second number and subtract it in tens and units, as below:

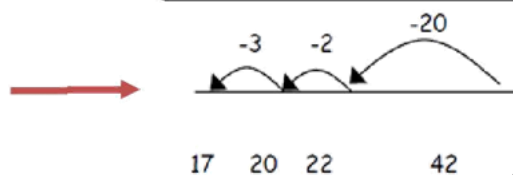


Move towards more efficient jumps back, as below:



Combine methods with use of a hundred square to reinforce understanding of number value and order.

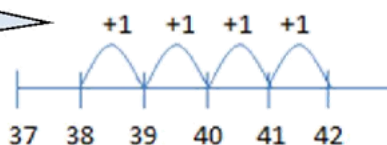
Teaching children to bridge through ten can help them to become more efficient, for example $42-25$:



Mental strategy - subtract numbers close together by counting on:

Start with the smaller number and count on to the largest.

$$42 - 38 = 4$$



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction.

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units*

Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Subtraction

Year 3 Subtracting with 2 and 3-digit numbers.

Introduce **partitioned column subtraction** method.

Step 1: introduce this method with examples where **no regrouping** is required.

$$\begin{array}{r} 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$

When learning to regroup, explore partitioning in different ways so pupils understand that when you regroup, the **VALUE** is the same i.e. $72 = 70+2 = 60+12 = 50+22$ etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce "regrouping" through practical subtraction. Make the larger number with Base 10 or Numicon, then subtract 47 from it.

$$72 - 47$$



$$\begin{array}{r} 60 + 12 \\ - 40 + 7 \\ \hline 20 + 5 = 25 \end{array}$$

Before subtracting '7' from the 72, they will need to regroup a 10 rod for 10 units, or move the 10 Numicon piece to the units so there are 12 in the units to subtract the 7 from. This will leave 60 in the tens to subtract the 40 from

STEP 3: Once pupils are secure with the understanding of "regrouping", they can use the partitioned column method to subtract any 2 and 3-digit numbers.

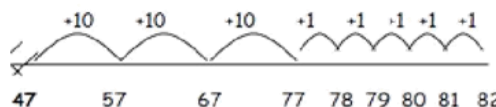
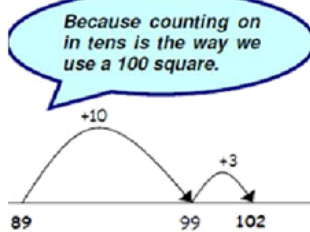
$$\begin{array}{r} 238 - 146 = 92 \\ \begin{array}{r} 100 \\ 200 + 30 + 8 \\ - 100 + 40 + 6 \\ \hline 0 + 90 + 2 \end{array} \end{array}$$

Subtracting money: partition into e.g. £1 + 30p + 8p

Counting on as a mental strategy for subtraction:

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

- Start at the smaller number and count on **in tens first**, then count on in units to find the rest of the difference:



Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units regroup, decrease, hundreds, value, digit*

Key skills for subtraction at Y3:

- Subtract mentally a: **3-digit number and ones**, **3-digit number and tens**, **3-digit number and hundreds**.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

Approximate,
Calculate
Check it!

Subtraction

Year 4 Subtract with up to 4-digit numbers

Partitioned column subtraction with "regrouping" (decomposition):

$$\begin{array}{r}
 2754 - 1562 = 1192 \\
 \hline
 2000 + \cancel{700} + 50 + 4 \\
 - 1000 + 500 + 60 + 2 \\
 \hline
 1000 + 100 + 90 + 2
 \end{array}$$

As introduced in Y3, but moving towards more complex numbers and values. Use **Dienes** and **Numicon**, to reinforce "regrouping".

Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction

$$\begin{array}{r}
 2754 \\
 - 1562 \\
 \hline
 1192
 \end{array}$$

To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method.

E.G. 2001-1789 is easier to do mentally or using a number line jotting to

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Approximate
Calculate
Check it!

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units, regroup, decrease, hundreds, value, digit,*

inverse

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Subtraction

Year 5 Subtract with at least 4-digit numbers

including money, measures, and decimals.

Compact column subtraction

(with "regrouping").

$$\begin{array}{r} \cancel{2}^{\text{'0}}\cancel{1}^{\text{'0}}\cancel{0}^{\text{'0}}\cancel{8}^{\text{'0}}6 \\ - \quad 2128 \\ \hline 28,928 \end{array}$$

Subtracting with larger integers.

Number lines continue to be used for addition and subtraction of time intervals

$$\begin{array}{r} \cancel{7}^{\text{'0}}\cancel{1}^{\text{'0}}\cancel{6}^{\text{'0}}\cancel{9}^{\text{'0}}.0 \\ - \quad 372.5 \\ \hline 6796.5 \end{array}$$

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Create lots of opportunities for subtracting and finding differences with money and measures.

Add a "zero" in any empty decimal places to aid understanding of what to subtract in that column.

Approximate,
Calculate,
Check it mate!

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units regroup, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal*

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps 1, 10, 100, 1000, 10 000, 100 000 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Subtraction

Year 6

Subtracting with increasingly large and more complex numbers and decimal values.

$$\begin{array}{r} \cancel{9}^{\text{M}} \cancel{8}^{\text{H}} \cancel{1}^{\text{K}}, 699 \\ - \quad 89,949 \\ \hline 60,750 \end{array}$$

Using the compact column method to subtract more complex integers

$$\begin{array}{r} \cancel{1}^{\text{kg}} \cancel{0}^{\text{kg}} 5 \cdot \cancel{4}^{\text{kg}} 19 \text{ kg} \\ - \quad 36 \cdot 08 \text{ kg} \quad \text{0 kg} \\ \hline 69 \cdot 339 \text{ kg} \end{array}$$

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

When working with measures it is important to record the units being used

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems. Using the inverse is important to check answers are correct and sensible.

Approximate,

Calculate,

Check it.

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count*

back, how many left, how much less is_? difference, count on, strategy, partition, tens, units decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal, regroup

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals
- across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Multiplication

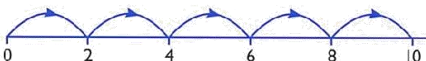
EYFS: Solve practical problems that involve doubling

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities and through discussion they will begin to solve problems involving doubling.



'Three apples for you and three apples for me. How many apples altogether?'



Year 1 Multiply with concrete objects, arrays and pictorial representations.



How many legs will 3 teddies have?

$$2 + 2 + 2 = 6$$



There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: *groups of, lots of, times, array, altogether, multiply, count*

Key skills for multiplication at Y1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

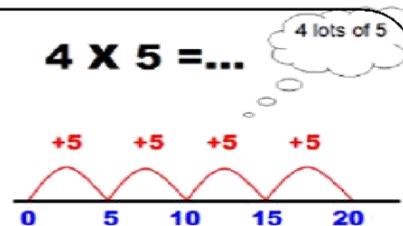
Multiplication



Year 2 Multiply using arrays and repeated addition and scaling up (using at least 2s, 5s, 10s)

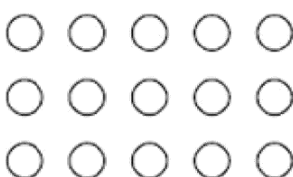
Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using \times and $=$ signs.



$$4 \times 5 = 20$$

Use arrays:



$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

$$5 \times 3 = 3 + 3 + 3 + 3 = \underline{15}$$

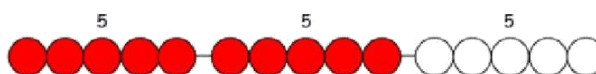
$$3 \times 5 = 5 + 5 + 5 = \underline{15}$$

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\quad} = 6$.

Use practical apparatus:

Such as bead strings

$$5 \times 3 = 5 + 5 + 5$$



Use mental recall:

- Children should begin to **recall multiplication facts** for 2, 5 and 10 times tables through practice in counting and understanding of the operation.

Key vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...*

Key skills for multiplication at **Y2**:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the \times and $=$ signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

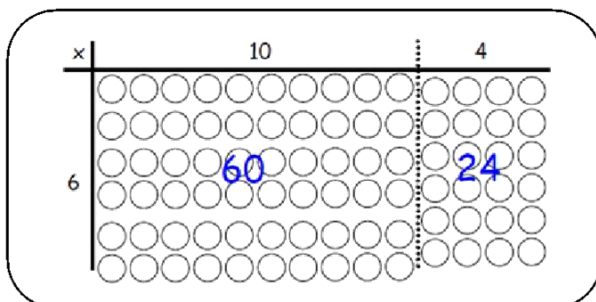
Multiplication



Year 3 Multiply 2-digits by a single digit number

Introduce the **grid method** for multiplying 2-digit by single-digits:

Link the layout of the grid to an array initially:



Eg. $23 \times 8 = 184$

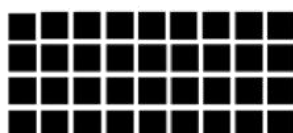
X	20	3
8	160	24

$$160 + 24 = 184$$

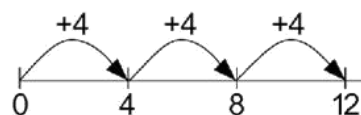
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s using Dienes or Cuisenaire rods), then translate this to grid method format.

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



$$9 \times 4 = 36$$



Key vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value*

Key skills for multiplication:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g. using commutativity ($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems $\square \times 5 = 20$, $3 \times \square = 18$, $\square \times \square = 32$

Multiplication



Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to **12 x 12**

Developing the grid method:

Eg. $136 \times 5 = 680$

X	100	30	6
5	500	150	30

500

150

+ 30

680

Encourage column addition to add accurately.

A few children may move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, and are already confident in "regrouping" for written addition.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g.:

346×9 is approximately $350 \times 10 = 3500$

Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables **up to 12 x 12**

Approximate,
Calculate,
Check it!

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Multiplication



Year 5 Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
- Children need to be taught to approximate first, e.g. for 72×38 , they will use **rounding**: 72×38 is approximately $70 \times 40 = 2800$, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28

327
x 4
28 (4x7)
80 (2x20)
1200 (4x300)
1308

327
x 4
1308

Pupils could be asked to work out a given calculation using the grid, and then compare it to "your" column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits

	10	8
10	100	80
3	30	24



	1	8
x	1	3
	5	4
1	8	0
2	3	4

18×3 on the 1st row

($8 \times 3 = 24$, regrouping the 2 for twenty, then 10×3 , adds the 2 tens).

18×10 on the 2nd row. say 8×10 , and 10×10 .

Moving towards more complex numbers:

1234
x 16
7404 (1234x6)
12340 (1234 x 10)
19744

3652
x 8
29216

Make sure the regroup digits are

Approximate,
Calculate,
Check it mate!

The grid can be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, "carry"**

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**. Solve problems where larger numbers are split into their factors. E.g. $18 \times 5 = 3 \times 6 \times 5$
- Multiply and divide integers and decimals by 10, 100 and 1000 Recognise and use square and cube numbers and how they are written
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Multiplication

Year 6 Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

Remind children that the single digit belongs in the unit's column.

Line up the decimal points in the question and the answer.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits** by a **single digit**; to multiply money and measures, and to **multiply decimals** with up to 2d.p. by a single digit.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits** by a 2-digit number.

$$\begin{array}{r} 1324 \\ \times 16 \\ \hline 7944 \\ 13240 \\ \hline 21184 \end{array}$$

Approximate,

Calculate,

Check it mate!

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, "carry", **tenths**, **hundredths**, **decimal**

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to **12×12** (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit \times 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

Division

EYFS Group, share and halve a number of objects

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities and through discussion they will begin to solve problems involving halving and sharing.

6 cakes are shared equally between 2 children. How many cakes do they have each? **SHARING**



There are 6 cakes, how many children can have 2 each? **GROUPING**



Year 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

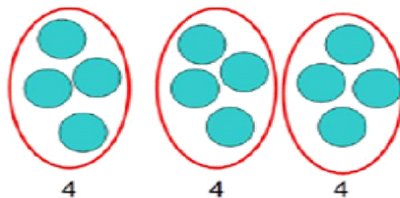
Grouping:

How many groups of 4 can be made with 12 stars? = 3



Sharing:

12 shared between 3 gives 4 each.



Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement.
"18 shared between 6 people gives you 3 each."

Pupils should:

Use lots of practical apparatus, arrays and picture representations

Be taught to understand the difference between "grouping" objects (How many groups of 2 can you make?) and "sharing" (Share these sweets between 2 people)

Be able to count in multiples of 2s, 5s and 10s.

Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary: *share, share equally, one each, two each..., group, groups of, lots of, array*
Key number skills needed for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

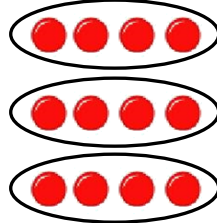
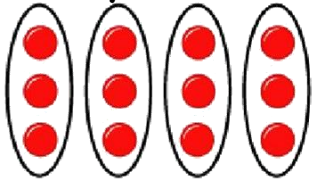
Division

Year 2 Group and share, using the \div and $=$ sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Relate division to multiplication facts

Arrays:



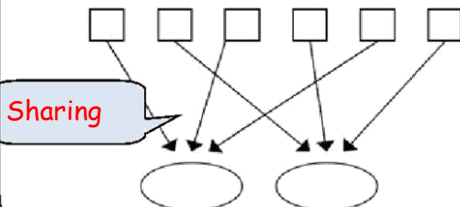
This represents $12 \div 3$, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent $12 \div 4 = 3$ if grouped horizontally.

Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?

Grouping



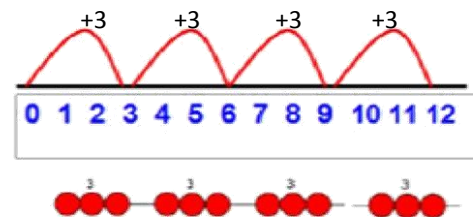
There are 6 sweets, how many people can have 2 sweets each?



Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "How many groups of _ in _?" Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" This is an important method to develop understanding of division as grouping.



$$12 \div 3 = 4$$

Instead of saying "12 divided by 3", say "how many groups of 3 are in 12?"

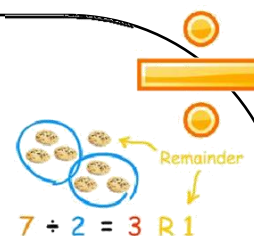
Key Vocabulary: *share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over*

Key number skills needed for division at Y2:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Division

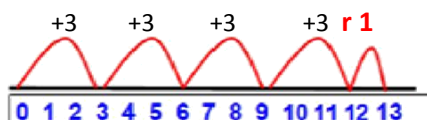
Year 3 Divide 2-digit numbers by a single digit (initially where there is no remainder in the final answer and moving on to this)



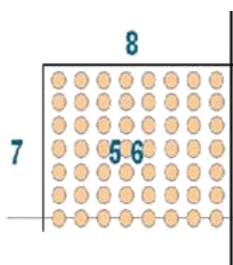
Grouping on a number line:

How many 3's in 13

$$13 \div 3 = 4 \text{ r } 1$$

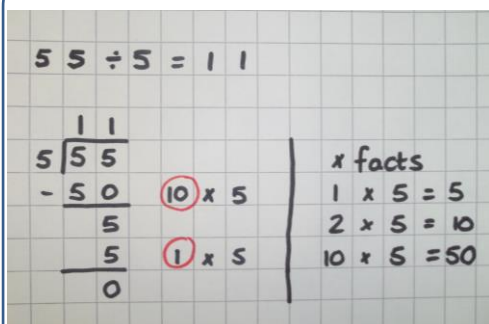


STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s



STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of division by comparing it to an array.

Chunking method:



STEP 3: The next step is to introduce the chunking layout which will be used increasingly as the children go further through the school. This method relies on children knowing and being able to work out key multiplication facts. These are then subtracted from the number being divided.

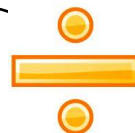
Real life contexts need to be used routinely to help pupils gain a full understanding and how to apply division

Key Vocabulary: *share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple*

Key number skills needed for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to for-mal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Division



Year 4 Divide up to 3-digit numbers by a single digit

(without remainders initially)

Initially working without remainders and using 2 digit numbers before progressing to 3 digit numbers and calculations with remainders.

Continue to chunking methods of division:

Developing efficiency when using chunks

$$84 \div 3 = 28$$

$$\begin{array}{r} 28 \\ 3 \overline{)84} \\ - 30 \\ \hline 54 \\ - 30 \\ \hline 24 \\ - 24 \\ \hline 00 \end{array}$$

10×3
 10×3
 8×3

x facts

$$\begin{array}{l} 1 \times 3 = 3 \\ 2 \times 3 = 6 \\ 5 \times 3 = 15 \\ 10 \times 3 = 30 \end{array}$$

$$\begin{array}{r} 28 \\ 3 \overline{)84} \\ - 60 \\ \hline 24 \\ - 24 \\ \hline 0 \end{array}$$

20×3
 8×3

x facts

$$\begin{array}{l} 1 \times 3 = 3 \\ 2 \times 3 = 6 \\ 5 \times 3 = 15 \\ 10 \times 3 = 30 \\ 20 \times 3 = 60 \end{array}$$

Real life contexts

need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

This year is spent introducing and consolidating informal and chunking methods of division.

When moving to working with 3 digit numbers it is important to consider the size of the chunks being subtracted and to aim to complete the calculation in as few steps as possible.

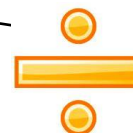
Time is also spent considering when the answers to division problems need rounding up or down depending on the context of the problem.

Key Vocabulary: *share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor*

Key number skills needed for division at Y4:

- Recall multiplication and division facts for all numbers up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division



Year 5 Divide up to 4 digits by a single digit, including those **with remainders**.

Chunking: Chunking is developed for larger numbers, including remainder answers:

Handwritten work for $356 \div 7 = 50 \text{ r } 6$. It shows a subtraction grid where 356 is divided by 7, resulting in 50 with a remainder of 6. To the right, a list of multiplication facts for 7 is provided: $1 \times 7 = 7$, $2 \times 7 = 14$, $10 \times 7 = 70$, $20 \times 7 = 140$, and $50 \times 7 = 350$.

As the numbers being divided increase, it becomes even more important to consider the size of the chunks being subtracted - multiples of 10, 20, 50 and 100 are encouraged where appropriate

Long Division: This method may be introduced for working with dividing a 4 digit number by a single digit number

Short Division: This method may be introduced to some pupils. (see Y6)

Handwritten short division for $4935 \div 8$. The quotient is 616 with a remainder of 7. The steps show 8 dividing into 49 to get 6, then 8 dividing into 13 to get 1, then 8 dividing into 55 to get 6, and finally 8 dividing into 75 to get 9 with a remainder of 7.

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, inverse, **quotient**, **prime number**, **prime factors**, **composite number (non-prime)**

Key number skills needed for division at Y5:

- Recall multiplication and division facts for all numbers up to 12×12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

Division

Year 6 Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

Introduce **long division by chunking** for dividing by 2 digits.

$$972 \div 36 = 27$$

$ \begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 152 \\ \underline{- 180} \\ 72 \\ \underline{- 72} \\ 0 \end{array} $	20×36 5×36 2×36	X facts $1 \times 36 = 36$ $2 \times 36 = 72$ $5 \times 36 = 180$ $10 \times 36 = 360$ $20 \times 36 = 720$
---	--	---

Pupils are encouraged to use larger chunks, such as 10 lots, 20 lots, 50 lots or 100 lots to minimise the number of steps needed to complete the calculation

Long division, as shown in Y5 may be extended to dividing by a 2 digit number, but only when pupils are secure with other methods.

Short division, for dividing by a single digit: e.g. $4935 \div 8$

$$\begin{array}{r}
 0616r7 \\
 8 \overline{) 4935}
 \end{array}$$

Must be aligned in place value for subtracting.

Short division techniques may also be introduced and further developed to work with increasingly larger numbers and using decimals.

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Approximate,
Calculate,
Check it mate!

Key Vocabulary: *As previously, & common factor*

Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12×12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

